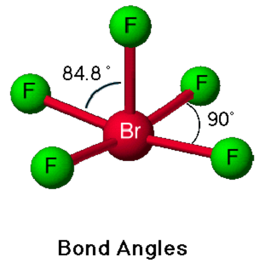
Exam Symmetry in Physics

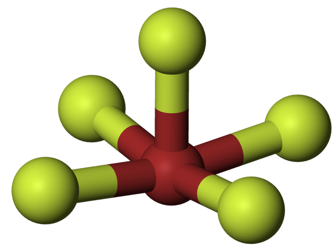
Date	April 2, 2023
Time	15:00 - 17:00
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- All subquestions (a, b, c) of the three exercises have equal weight
- Illegible answers will be not be graded
- Good luck!

Exercise 1

Consider the Bromine pentafluoride molecule BrF_5 depicted in the figures and its symmetry group G_B.





Note that four F atoms form a square in a plane and the axis through the fifth F atom and the Br atom goes through the center of that square.

(a) Identify the eight symmetry transformations that leave this molecule invariant and divide them into conjugacy classes, using geometrical arguments.

(b) Construct the character table of G_B and explain how the entries are obtained.

(c) Determine the characters of the three-dimensional vector representation D^V of G_B and use them to determine whether the molecule can have an electric dipole moment.

Exercise 2

Consider the electrostatic Maxwell stress tensor (i, j can take the values 1, 2, and 3):

$$\sigma_{ij} = \varepsilon_0 E_i E_j - \frac{1}{2}\varepsilon_0 \vec{E}^2 \delta_{ij},$$

where ε_0 denotes the electric constant (vacuum permittivity) and $\vec{E}$ the electric field.

(a) Isolate the parts of the stress tensor that transform independently from each other under $SO(3)$ transformations, i.e. under the group of rotations in three dimensions, and indicate under which irreducible representation(s) of $SO(3)$ these parts transform, together with the corresponding dimension(s).

(b) Explain why only rank-2 tensors that are proportional to the identity are invariant under $SO(3)$ transformations. Recall that tensors of rank 2 transform according to $\sigma' = D^V \sigma (D^V)^{-1}$.

(c) Indicate how the stress tensor σ_{ij} behaves under reflection in the origin (space inversion) and how it behaves under this reflection if the electric field is replaced by a magnetic field (the magnetostatic stress tensor).

Exercise 3

Consider the group $SO(2)$ of rotations around the z-axis in three dimensions. Consider its action on the angular momentum states $|l, m\rangle$ through the operator

$$U(R_z) = \exp\left(-\frac{i}{\hbar}\theta\, L_z\right), \tag{1}$$

where $R_z = R_z(\theta)$ is a rotation over an angle θ around the z-axis.

(a) Write down the explicit matrix for L_z acting on the space of $|l, m\rangle$ states with $l = 2$.

(b) Write down the explicit matrix $D^{(l=2)}(R_z)$ formed by matrix elements $\langle 2, m|U(R_z)|2, m'\rangle$.

(c) Show or argue that this representation of $SO(2)$ is reducible and write down the complex irreps of $SO(2)$ that it is composed of.